2021-2022

V. Simmons

Progression of Calculations

Bentley new village primary school

**CONTENTS :**

**Page Section**

1 Aims of Policy

5 Mathematical Language

6 Overview of Approaches by Year Group

9 Progression of Calculations

10 Addition

27 Subtraction

42 Multiplication

59 Division

70 Glossary

**Aims:**

At Bentley New Primary we aim to develop pupils’ calculation skills through the provision of practical, oral, mental and written mathematical activities. Pupils will learn how to use a range of models and images to support their mental and informal written methods of calculation, leading to the development of efficient written methods as the children move through school. Teachers will present their classes with opportunities to develop their understanding of all 4 operations using the concrete, pictorial and abstract.

The 2014 National Curriculum provides a structured and systematic approach to the teaching of calculation. The aim is for mental calculations and written procedures to be performed efficiently, fluently, and accurately with understanding. Procedures and understanding are to be developed in tandem. End of key stage expectations are explicit in the programme of study. At Bentley New Village Primary School, we have a consistent approach to the teaching of written calculation methods in order to ensure continuity and progression across the school.

**Age related expectations and Mastery:**

Although the National Curriculum gives guidance as to the expectations for the teaching of written methods, **it is vital that pupils are taught according to the stage that they are currently working at,** being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, pupils must not simply rote-learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. We focus on all children achieving what is expected of their age group and not going beyond this. Evidence shows that children need to be able to understand a concept, apply it in a range of situations and then be creative to really understand it. Simply going beyond their age group does not guarantee they understand something, it just means they have heard it.

Please note that the principle of the concrete-pictorial-abstract (CPA) approach is that for pupils to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, ‘proving’ something or ‘working out’ could involve use of the concrete or pictorial. In short, these are not always ‘exclusive’ representations.

At our school no child will be taught content from the year group above them, they will spend time becoming true masters of content, applying and being creative with new knowledge and skills in multiple ways.

**Purpose:**

This policy makes teachers aware of the written strategies that children are formally taught as they progress through school. The policy only details the strategies - teachers must plan opportunities for children to apply these; for example, when solving problems, developing reasoning skills or where opportunities emerge elsewhere in the curriculum. **Teachers should provide a context for calculation.**  It is important that any type of calculation is given a real life context or problem solving approach to help build children’s understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods. It is also important for children to be confident to use mental and written strategies to explain their thinking**.** This must be a priority within calculation lessons**.** Written methods need to be viewed as tools to enable children to solve problems and record their thinking in an organised way. This guide is not prescriptive and is based on suggested calculations from the national curriculum (2014). The following methods are organised by year group though should be used flexibly according to individual need and ability.

**Before carrying out a calculation, children will be encouraged to consider:**

* Can I do it in my head? (using rounding, adjustment) Secure mental methods which are developed from early years
* The size of an approximate answer (estimation). Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.
* Could I use jottings to keep track of the calculation? Visual models and images including number lines and arrays and move on to experience of expanded methods to develop understanding and avoid rote-learning
* Which resources to use to support their calculation. The development of an efficient written calculation relies on practical hands-on experience including a range of manipulatives

**Monitoring and evaluation**

The teaching of calculations will be monitored using evidence collected from:

* Planning scrutiny
* Classroom observations
* Scrutiny of children’s work
* AFL opportunities within lessons
* Discussions with children
* Evidence sheets

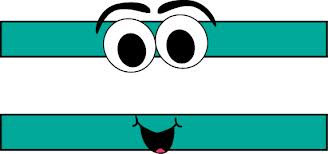
**Mathematical Language**

|  |  |
| --- | --- |
|  |  |
| ones | units |
| is equal to | equals |
| zero | oh (the letter O) |

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct, for example, using the term ‘regroup’ rather than ‘carry’ or ‘borrow.’

For all calculations we need children to practically carry out the calculation (action), say what they have done (language) and then record this. Therefore, it is extremely important that children are introduced to the mathematical language first and practise using this to explain their actions, thinking and the reasoning behind the strategy they have used.

Before children move on to formal methods, which are outlined in the policy, the recoding should always be children’s own concept to begin with. Children will need to use the mathematical language to explain their workings and understanding.



**Equals**

Is the value same as

Is equivalent to

****

**Division**

Share equally

Divide by

Group

Halve, quarter etc

Chunk

Multiple

Pairs

Remainders

Array

****

**Multiply**

Sets of

Lots of

Groups of

Times

Multiply by

Double – Twice, three times etc

Product

Multiple

Factor

Repeated addition

****

**Subtraction**

Take away

Minus

Decrease

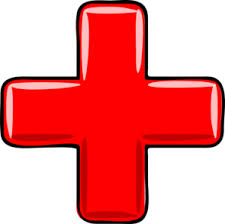
Less than

Difference

How much more

Subtract

Left

** Addition**

Add

And

Plus

Increase

More than

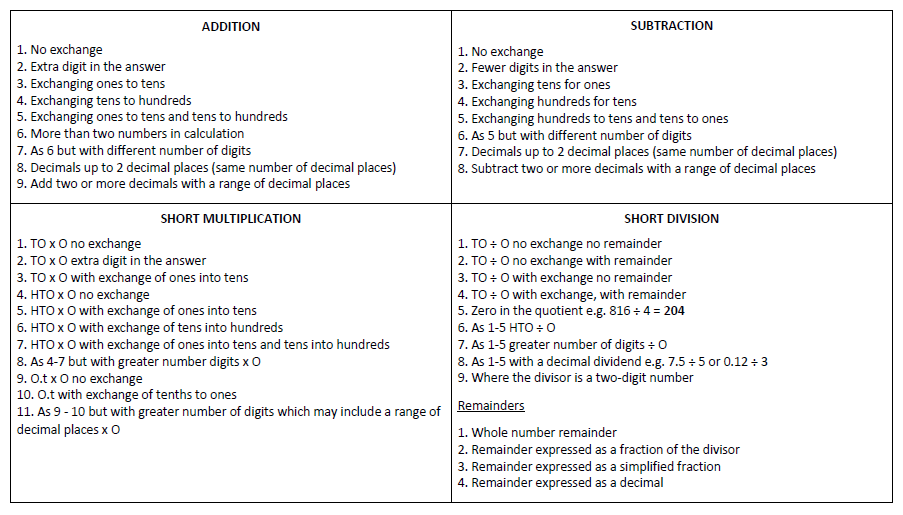
Total

Altogether

Sum of

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Overview of Calculations** | | | | | | | |
|  | ***EYFS*** | | ***YEAR 1*** | ***YEAR 2*** | ***YEAR 3*** | ***YEAR 4*** | ***YEAR 5*** | ***YEAR 6*** |
| **Addition** | Rhymes and songs, practical, mark making, concrete and physical representations. Draw objects | * Subitising * Count all | * Count all * Number bonds * Counting on * Regrouping ten ones to make 10 * Adding 1,2, 3 more * Adding three single digits (make ten first) * Adding multiples of ten * Part whole model * Make 10 strategy * Partitioning to add (no regrouping) | * Part whole model * Make 10 strategy * Partitioning to add no regrouping * Column Method – no regrouping. Some exchange * Counting on in tens and hundreds * Using known facts to create derived facts * Partitioning one number then adding tens and ones * Round and adjust | * Column method with regrouping (up to 3 digits) * Round and adjust * Add and subtract mentally * Find 10, 100 more | * Column method with regrouping (up to 4 digits) * Round and adjust * Calculating decimal numbers * Count forwards and backwards in steps of 10, 100 and 1000 * Use known facts and knowledge of place value to derive facts * Add and subtract multiples of 10, 100 and 1000 * Adding and subtracting by partitioning one number * Near doubles * Partition both numbers and combine | * Column method with regrouping (with more than 4 digits) * Column method – decimals – with the same amount of decimal places * Round and adjust * Count forwards and backwards in steps of 10, 100 and 1000 * Use known facts and knowledge of place value to derive facts * Add and subtract multiples of 10, 100 and 1000 * Adding and subtracting by partitioning one number * Near doubles * Partition both numbers and combine | * Column method with regrouping * Column method – decimals – with different amounts of decimal places * Round and adjust * Count forwards and backwards in steps of 10, 100 and 1000 * Use known facts and knowledge of place value to derive facts * Add and subtract multiples of 10, 100 and 1000 * Adding and subtracting by partitioning one number * Near doubles * Partition both numbers and combine |
| **Subtraction** | * Taking away from ones | * Taking away ones * Part Part whole model * Regroup into 10 ones * Taking away from the tens * Subtracting multiples of ten * Partitioning to subtract without regrouping * Counting back * Make 10 | * Column method with regrouping * Partitioning to subtract without regrouping * Counting back * Make 10 * Subtracting tens and one * Find the difference, * Using known number facts to create derived facts * Round and adjust * Column method- no regrouping. Some exchange | * Column method with regrouping (up to 3 digits) * Using known number facts to create derived facts * Round and adjust * Add and subtract mentally | * Column method with regrouping (up to 4 digits) * Using known number facts to create derived facts * Round and adjust * Add and subtract mentally * Count forwards and backwards in steps of 10, 100 and 100. * subtracting by partitioning one number and applying facts | * Column method with regrouping (with more than 4 digits) * Column method – decimals – with the same amount of decimal places * Using known number facts to create derived facts * Round and adjust * Add and subtract mentally * subtracting by partitioning one number and applying facts * Calculate by counting back * Calculate by counting on | * Column method with regrouping * Column method – decimals – with different amounts of decimals places * Using known number facts to create derived facts * Round and adjust * Add and subtract mentally * subtracting by partitioning one number and applying facts * Calculate by counting back * Calculate by counting on |
| **Multiplication** | * Grouping – making equal groups | * Equal groups * Repeated addition * Skip counting * Doubling | * Repeated addition * Skip counting * Doubling * Arrays * Multiplication is commutative * Part Part whole * Include x symbol | * Skip counting * Doubling * Multiplication is commutative * Part Part whole * Multiplying by 10 and 100 * Using known facts for multiplying multiples of 10 and 100 * Multiplication of 2 digit numbers with partitioning (no regrouping) * Multiplication of 2-digit numbers with partitioning (regrouping) | * Doubling and halving * Multiplying by 10 and 100 * Using known facts for multiplying multiples of 10 and 100 * Multiplying by partitioning one number and multiplying each part * Mental multiplication of three 1 digit numbers using the associative law * Short multiplication * Grid method (as a step towards long multiplication) | * Doubling and halving * Decimals * Multiplying by 10, 100 and 1000 * Using known facts for multiplying multiples of 10 and 100 * Short multiplication * Multiply by partitioning one number and multiplying each part * Using knowledge of factors * Long multiplication | * Doubling and halving * Short multiplication * Decimals * Multiplying by 10, 100 and 1000 * Using known facts for multiplying multiples of 10 and 100 * Multiply by partitioning one number and multiplying each part * Using knowledge of factors * Long multiplication |
| **Division** | * Sharing * Division as grouping | * Sharing * Division as grouping | * Division as sharing * halving * Division as grouping * Part part whole model * Division within arrays * Bar model * Division with remainder | * Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down | * Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down * Derived facts * Short division * Division of a one or 2 digit number by 10 or 100 | * Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down * Derived facts * Short division * Doubling and halving * Using knowledge of multiples to divide * Using knowledge of factors to divide * Long division | * Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down * Derived facts * Short division * Doubling and halving * Using knowledge of multiples to divide * Using knowledge of factors to divide * Long division |

Progression of calculations:



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Addition** | | | | |
| ***Suggested* Year** | **Strategies** | **Concrete** | **Pictorial** | **Abstract** |
| EYFS | Children will have been introduced to calculation through a range of counting activities songs, rhymes and games, with the emphasis on practical, hands-on experiences. Children will have been introduced to the terms more and less practically before any number knowledge has been introduced. This is an essential step before teaching calculation | | | |
| EYFS | **Subitising** | Children need to recognise a small group of objects/dots/fingers as the total without counting them | Use a range of resources e.g. stones before moving to recording | Pupils recognise amounts without counting. |
| EYFS &  Year 1 | **Count all** –  *Joining two groups and then recounting all objects using one-to-one correspondence* |  | C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].png  Image result for part whole model addition | 4 + 3 = 7  10= 6 + 4  5 + 3 = 8 |
| Year 1 | **Number Bonds** |  | [https://s-media-cache-ak0.pinimg.com/236x/a4/11/07/a411079575f7fc4d1b122046baf40638.jpg](https://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0ahUKEwin2K6fxdDNAhUIIsAKHfmvDGEQjRwIBw&url=https://www.pinterest.com/drnicki7/number-bonds/&psig=AFQjCNGa5inWzvjCRyWQSxsU6bn2gZSrBQ&ust=1467403244819435) | 1 + \_\_\_\_\_ = 10  10 = 9 + 1  How many different ways can you make 10? |
| Year 1 | **Counting on**  *As a strategy, this should be limited to adding small quantities only (1, 2, or 3) with pupils understanding that counting on from the greater number is more efficient* |  | 8  1 | 8 + 1 = 9  15 = 12 + 3  5 plus 3 = 8  3 more than 5 = 8 |
| Year 1 | **Re-grouping ten ones to make 10**  *This is an essential skill that will support column addition later.* |  |  | 3 + 9 = |
| Year 1 | **Adding 1, 2, 3 more**  *Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as;*  *‘1 more than 5 is equal to 6.’*  *‘2 more than 5 is equal to 7.’*  *‘8 is 3 more than 5.’* |  |  | 1 more than 5  5 + 1 = 6  2 more than 5  5 + 2 = 7 |
| Year 1 | **Adding three single digits (make ten first)**  *Pupils may need to try different combinations before they find the two numbers that make 10.* | *The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.*  *The second bead string shows 4, 6 and then 7.*  *The final bead string shows how they have now been put together to find the total.* | C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].png  Add together three groups of objects. Draw a picture to recombine the groups to make 10.  +  + | Combine the two numbers that make 10 and then add on the remainder. |
| Year 1 | **Adding multiples of 10**  *Using the vocabulary of 1 ten, 2 tens, 3 tens etc.* ***alongside*** *10, 20, 30 is important, as pupils need to understand that it is a* ***ten*** *and not a one that is being added and they need to understand that a ‘2’ digit in the tens column has a value of twenty. It also emphasises the link to known number facts. E.g. ‘2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.* |  |  | 3 tens + 5 tens = \_\_\_\_\_\_\_ tens  30 + 50 =  36 + 40 = |
| Year 1  & 2 | **Part-Part-whole**  *Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.*  *This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.*  *Y2 - Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.* |  | Image result for part whole model | 10 = 6 + 4  10 − 6 = 4  10 − 4 = 6  10 = 4 + 6  5  3  Use the part-part whole diagram as shown above to move into the abstract. |
|  |  |  |
| Year 1 – Year 2 | **‘Make 10’ strategy**  *Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.*  *The colours of the beads on the bead string make it clear how many more need to be added to make ten.*  *Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.*  *Y2 - How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.*  *It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.* |  |  | 6 + 5 = 11  7 + 4= 11  If I am at seven, how many more do I need to make 10. How many more do I add on now? |
|  |  |  |
| Year 1-Year 2 | **Partitioning to add (no regrouping)**  When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.  *Y2 - As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3-digit numbers but do not record a formal written method if there is no regrouping.* | Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure. | | 24 + 13 = 37  *4 + 3 = 7*  *20 + 10 = 30*  *30 + 7 = 37* |
| 24 + 15=  Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. | After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  T O |  |
| Year 1-Year 6 | **Introducing column method for addition, regrouping only**  ***Year 1*** *- Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.*  ***Year 2*** *-* *Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.*  **Year 3 & 4** - *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it.*  *Asking them ‘Can you think of a more efficient method?’ will challenge them to apply their number sense / number facts*  *to use efficient mental methods where possible.*  *In Year 3 they become more familiar with calculations that require ‘regrouping to regroup’. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.*  *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.*  *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.*  ***In Year 5 & 6****, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.*  *Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.*  *Continue to use concrete manipulatives alongside the formal method.*  *When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.* |  |  | **Introducing column method for addition, regrouping only**  *Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.* |
|  | |  |
| As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one.  Regrouping (including multiple separate instances)  672 + 136 734 – 82  468 + 67 831 - 76  275 + 386 435 – 188  ‘Regrouping to regroup’  204 – 137  1035 - 851 | | |
| Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.    Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.  This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.  As children move on to decimals, money and decimal place value counters can be used to support learning. | Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. | Start by partitioning the numbers before moving on to clearly show the exchange below the addition.  As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. |
| Year 2 | **Counting on in tens and hundreds** |  |  |  |
| Year 2 | **Using known facts to create derived facts**  *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.* |  | | |
| Year 2 | **Partitioning one number, then adding tens and ones**    *Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.* |  |  | 22 + 17 = 39 |
| Year 2 – Year 6 | **Round and adjust (sometimes known as a compensating strategy)**  *Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.*  *Year 3 onwards - Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.*    *It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.*  *Build flexibility by completing the same calculation in a different order.* |  |  | 22 + 17 = 39 |
|  | |  |
|  | |  |
| Year 3 | **Add and subtract numbers mentally, including:**   * **a three-digit number and ones;** * **a three-digit number and tens;** * **a three-digit number and hundreds**   *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* | It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images.  No regrouping  345 + 30 274 - 50  1128 + 300 1312 - 300  326 + 342 856 - 724    I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375.  With some regrouping  416 + 25 232 - 5  383 + 130 455 - 216  611 + 194 130 - 40  1482 + 900 2382 - 500 | | |
| Year 3 | **Find 10, 100 more or less than a given number**  *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds including use of concrete manipulatives and appropriate images.* |  | | |
| Year 4 | **Calculating with decimal numbers**  *Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.*  *Place value counters are another useful manipulative for representing decimal numbers.* |  | |  |
| Year 4 – Year 6 | **Count forwards and backwards in steps of 10, 100 and 1000.**  *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.*  *In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.*  *In Year 6 pupils work with numbers up to 10 000 000.* |  | Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.  E.g. 990 + 10 or 19.9 + 0.1 |  |
| Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed    Counting sticks | Number lines |  |
| Year 4 – Year 6 | **Using known facts and knowledge of place value to derive facts.**  **Add and subtract multiples of 10, 100 and 1000 mentally**  *In* ***Year 4*** *- Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000.*  *In* ***Year 5*** *extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.*  *In* ***Year 6*** *extend to multiples of one million.*  *These derived facts should be used to estimate and check answers to calculations.* |  | |  |
|  | | *Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.* |
| Year 4 – Year 6 | **Adding and subtracting by partitioning one number and applying known facts.**  *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*  *Pupils can use this strategy mentally or with jottings as needed.*  ***Year 5 and 6*** *- Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.*  *They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.* | See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery. | |  |
|  | |  |
| Year 4 – Year 6 | **Near doubles**  *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.*  *Year 5 & 6*  *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.* | 1600 + 1598 = double 1600 – 2 | |  |
| 160 + 170 = double 150 + 10 + 20  160 + 170 = double 160 + 10 or 160 + 170 = double 170 - 10  2.5 + 2.6 = double 2.5 + 0.1 | |  |
| Year 5 and 6 | **Partition both numbers and combine the parts**  *Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.*  *If multiple regroupings are required, then pupils should consider using the column method.* |  | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Subtraction** | | | | |
| ***Suggested* Year** | **Strategies** | **Concrete** | **Pictorial** | **Abstract** |
| EYFS/ Year 1 | **Taking away from the ones**  *When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.* | Use physical objects, counters, cubes etc to show how objects can be taken away.    4-2  6 – 2 = 4 | Cross out drawn objects to show what has been taken away.    Cross out drawn objects to show what has been taken away. | 18 - 3= 15 |
| Year 1 | **Part-Part-Whole Model**  *Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them.*  *Pupils start with ten cubes placed on the whole.*  *They then remove what is being taken away from the whole and place it on one of the parts.*  *The remaining cubes are the other part and also the answer. These can be moved into the second part space.* |  | Use a pictorial representation of objects to show the part part whole model. | If 10 is the whole and 6 is one of the parts. What is the other part?  10 - 6 =  10  5  Move to using numbers within the part whole model. |
| Year 1 | **Regroup a ten into 10 ones**  *After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method*. |  |  | 20 – 4 = |
| Year 1 | **Taking away from the tens**  *Pupils should identify that they can also take away from the tens and get the same answer.* |  |  | 15 – 9 = |
| Year 1 | **Subtracting multiples of ten**  *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a* ***ten*** *not a one that is being taken away.* |  |  | 6 tens – 2 tens = tens  60 – 20 =  38-10 = |
| KS1-KS2 | **Column method with regrouping**  *Y1 - This example shows how pupils should work practically when being introduced to this method.*  *There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.*  **Y2 -** *Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. 232 – 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not missed in the recording.*  *In* ***Year 5****, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.*  *Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.*  *Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.* | 34 – 17 = | |  |
| Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.  Make the larger number with the place value counters. Start with the ones. Can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.    Now I can subtract my ones.  Now look at the tens. Can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction    Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount. | Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. | Children can start their formal written method by partitioning the number into clear place value columns.    Moving forward the children use a more compact method. |
| The term regrouping should be the language used. You can use the terms exchange with subtraction but carefully. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 2 ones). Or you can ‘exchange’ one of the tens for 10 ones resulting in the 5 tens and 12 ones | |  |
| KS1 | **Partitioning to subtract without regrouping**  *Y1 - Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.*  *When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording.*  *Y2 - Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.* | Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first. | Draw the Base 10 or place value counters alongside the written calculation to help to show working. |  |
| http://media.showmeapp.com/files/205114/pictures/thumbs/1100814/last_thumb1379615590.jpg |
| KS1 | **Y1 Counting Back**  *Subtracting 1, 2, or 3 by counting back*  *Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.*  **Y2 Counting back in multiples of ten and one hundred** | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones eg 13 – 4 =    http://3.bp.blogspot.com/-mFqQPE4k1TE/VGzRNnUu30I/AAAAAAAAAJM/12p6qvgkmoE/s1600/EvenOdd_ColoredCounters_Scattered.jpgUse counters and move them away from the group, as you take them away counting backwards as you move them. | Count back on a number line or number track    Start at the bigger number and count back the smaller number showing the jumps on the number line.    This can progress all the way to counting back using two 2 digit numbers. | 13 – 4 =  Put 13 in your head, count back 4. What number are you at? Use your fingers to help. |
|  |  |  |
| KS1 | **Make 10**  *To subtract a 1-digit number from a 2-digit number.*  *Y1 - Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.*  *Y2 - How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.*  *It relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten.*  *Pupils should develop an understanding that the parts can be added in any order.* | 14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. | Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. | 16 – 8=  How many do we take off to reach the next 10?  How many do we have left to take off? |
|  |  |  |
| Year 2 | **Subtracting tens and ones**  *Pupils must be taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required.* |  |  | 53 − 12 = 41 |
| Year 2 | **Find the difference** | Image result for two towers of cubesCompare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference | Count on to find the difference.  http://image.slidesharecdn.com/intro-to-sm-1220840292402057-8/95/intro-to-singapore-math-13-728.jpg?cb=1345557040Draw bars to find the difference between 2 numbers. | Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. |
| Year 2 – Year 4 | **Using known number facts to create derived facts**  *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to add multiples of ten and 100.*  *Year 4 - Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.* |  | | |
|  | | |
| Year 2 - 6 | **Round and adjust (sometimes known as a compensating strategy)**  *Pupils must be taught to round the number that is being subtracted.*  *Y4+ - Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.*  *Build flexibility by completing the same calculation in a different order.* |  |  | 53 − 17 = 36 |
|  | |  |
|  | |  |
| Year 3-6 | **Add and subtract numbers mentally, including:**   * **a three-digit number and ones;** * **a three-digit number and tens;** * **a three-digit number and hundreds** | *See addition section* | |  |
| Year 4 | **Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000.**  *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.*  **Count forwards and backwards in tenths and hundredths** |  | Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.  E.g. 990 + 10 or 19.9 + 0.1 |  |
| Year 4 - 6 | **Adding and subtracting by partitioning one number and applying known facts.**  *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*  *Pupils can use this strategy mentally or with jottings as needed.*  *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.* | See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery. | |  |
| Year 5 and 6 | **Calculate difference by “counting back”**  *It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.* |  | |  |
| Year 5 and 6 | **Calculate difference by “counting on”**  *Addition strategies can be used to find difference.* |  | |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Multiplication** | | | | |
| ***Suggested* Year** | **Strategies** | **Concrete** | **Pictorial** | **Abstract** |
| EYFS/ Year 1 | **Making equal groups and counting the total**  *How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.* |  |  |  |
| KS1 | **Repeated addition**  *This strategy helps pupils make a clear link between multiplication and division as well as exemplifying the ‘repeated addition’ structure for multiplication. It is a natural progression from the previous ‘count all’ strategy as pupils can be encouraged to ‘count on’. However, as number bonds knowledge grows, pupils should rely more on these important facts to calculate efficiently.* | Use different objects to add equal groups. |  | Write addition sentences to describe objects and pictures.  3 + 3 + 3 = 9 |
|  |  |  |
| KS1 & Year 3 | **Y1 Skip counting in multiples of 2, 5, 10 from zero**  *The representation for the amount of groups supports pupils’ understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.*  **Y2 - Skip counting in multiples of 2, 3, 4, 5, 10 from zero**  *Dot arrays can be used to create a visual representation for the different multiplication facts.*  ***Y3* Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10** | *Count the groups as pupils are skip counting.* | Use a number line or pictures to continue support in counting in multiples.  *Pupils can use their fingers as they are skip counting.* | Count in multiples of a number aloud.  Write sequences with multiples of numbers.  2, 4, 6, 8, 10  5, 10, 15, 20, 25 , 30  4 lots of 2  4 groups of 2 |
|  |  |  |
|  | |  |
| KS1 & KS2 | Doubling  **Y2-3**  **Doubling to derive new multiplication facts**  *Pupils learn that known facts from easier multiplication tables can be used to derive facts from related times tables using doubling as a strategy.*  *At this stage they double the 2× table facts to derive the 4× table facts.* | Use practical activities to show how to double a number. | Draw pictures to show how to double a number. | Partition a number and then double each part before recombining it back together. |
|  | | |
|  | **Doubling and halving**  *Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.*  *Doubling and halving can then be used in larger calculations.* |  | |  |
| Year 2 | **Arrays to represent multiplication equations**  *Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.*  *Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.*  *The relationship between multiplication and division also begins to be demonstrated.* | http://www.australiancurriculumlessons.com.au/wp-content/uploads/2013/05/arrays-multiplication-division-lesson.jpgCreate arrays using counters/ cubes to show multiplication sentences. |  | Use an array to write multiplication sentences and reinforce repeated addition. |
| Year 2 & 3 | **Multiplication is commutative**  *Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.* |  |  | Draw arrays in different rotations to find **commutative** multiplication sentences.  If I know… then I know… |
| Year 2 &  Year 3 | **Use of part-part-whole model to establish the inverse relationship between multiplication and division**  *This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division.* | *Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts. It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.* |  | |
| KS2 | **Multiplying by 10 and 100**  *Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.*  *Y4 -* **Multiplying by 10 and 100**  *When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.*  *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.*  *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000*  **Y 5 & 6 Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**  *Avoid saying that you “add a zero” when multiplying by ten and instead use the language of place holder.* |  | |  |
|  | |  |
|  | |
| KS2 | **Using known facts for multiplying by multiples of 10 and 100**  *Pupils’ growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.*  *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*  *Year 4*  **Using known facts and place value for mental multiplication involving multiples of 10 and 100**  *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.*  *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger ‘fact families’ to be derived from a single known number fact.* |  | |  |
|  | |  |
|  | **Year 5 and 6**  *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger ‘fact families’ to be derived from a single known number fact.*  *Knowledge of commutativity is further extended and applied to find a range of related facts.*  *Pupils should work with decimals with up to two decimal places.*  *These derived facts should be used to estimate and check answers to calculations.* |  | |  |
| Year 3 | **Multiplication of 2-digit numbers with partitioning (no regrouping)**  *Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.*  *Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.* |  |  |  |
| Year 3 | **Multiplication of 2-digit numbers with partitioning (regrouping)**  *Using concrete manipulatives and later moving to using images that represent them, supports pupils’ early understanding, leading towards formal written methods in Year 4.*  *Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.* |  | |  |
| Year 4 | **Multiplying by partitioning one number and multiplying each part**  *As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.*  **Distributive law**  **a x (b + c) = a x b + a x c** |  | | |
| Year 4 | **Mental multiplication of three 1-digit numbers, using the associative law**  *Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.* |  | |  |
| Year 4 - 6 | **Short multiplication of 3-digit number by 1-digit number**  *To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.*  *With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.* | To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed | |  |
| Year 5 and 6 | **Multiply by partitioning one number and multiplying each part**  **Distributive law**  **a x (b + c) = a x b + a x c**  Build on pupils’ understanding of arrays of counters to represent multiplication to see that area models can be a useful representation: |  | |  |
| Year 5 and 6 | **Using knowledge of factors**  *In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.*  *Pupils will be using the commutative and associative laws of multiplication.*  **Commutative law**  **a x b = b x a**  **Associative law**  **a x b x c = (a x b) x c**  = a x (b x c)  *They should explore and compare the different options and choose the most efficient order to complete calculations.* |  | |  |
| Year 5 and 6 | **Multiplying by a 2-digit number**  **Formal written method of long multiplication**  *In Year 6 pupils are extended from multiplication by a 1-digit number to multiplication by a 2-digit number.*  *Extend the place value chart model used in Year 4, using an additional row on the place value chart.*  *Extend understanding of the distributive law to develop conceptual understanding of the two rows of the formal written method.*  *Dienes blocks can be used to construct area models to represent this.* |  | |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Division** | | | | |
| ***Suggested* Year** | **Strategies** | **Concrete** | **Pictorial** | **Abstract** |
| EYFS/ KS1 | **Sharing objects into groups**  *Pupils should become familiar with division equations through working practically.*  *The division symbol is not formally taught at this stage.*  **Division as grouping**  *Here, division is shown as grouping.*  *If we have ten forks and we put them into groups of two, there are 5 groups.* | I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities. | Share 9 buns between three people.  9 ÷ 3 = 3 |
|  |  | Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  http://gcamath3.weebly.com/uploads/9/1/4/0/9140392/200455_orig.jpg | 28 ÷ 7 = 4  Divide 28 into 7 groups. How many are in each group? |
| Year 1 and 2 | **Division as sharing**  *Here, division is shown as sharing.*  *If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.* |  |  |  |
| Year 2 | **Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication**  *Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.*  *They begin to use dot arrays to develop a more abstract concept of division.*  *It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.* |  |  | The whole is nine. There are three equal parts. What is the value of each part? |
| Year 2 | Division within arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15 | Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating four linking number sentences.  7 x 4 = 28  4 x 7 = 28  28 ÷ 7 = 4  28 ÷ 4 = 7 |
| Year 2 | Division with a remainder | 14 ÷ 3 =  Divide objects between groups and see how much is left over  Image result for counters | Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. | Complete written divisions and show the remainder using r.  http://amsi.org.au/teacher_modules/G7/G7_qt2%202.png |
| KS2 | **Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down**  *Pupils use the strategy of ‘scaling down’, representing numbers with concrete manipulatives and making the value ten times smaller*  **Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping**  *Pupils divide by 10, 100 and 1000 by making groups of the divisor.*  *Y4 -* **Dividing by 10 and 100**  *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.*  *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*  *Y5 AND 6 -*  **Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**  *Avoid saying that you “add a zero” when multiplying by ten and instead use the language of place holder.*  *Use place value counters and charts to visualise and then notice what happens to the digits.* | 3 × 10 = 30  30 ÷ 10 = 3 | | |
|  |  | |  |
| Year 4 – 6 | **Derived facts**  *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.*  *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.* |  | |  |
| **Deriving facts from known facts**  *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.*  *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.* |  | |  |
| Year 4 | **Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths**  *When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.* |  | |  |
| Year 5 and 6 | **Doubling and halving**  *Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.*  *Doubling and halving can then be used in larger calculations.* | See multiplication | |  |
| Year 5 and 6 | **Using knowledge of multiples to divide**  *Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).* |  | |  |
| Year 5 and 6 | **Using knowledge of factors to divide**  *Pupils explore this strategy when using repeated halving.*  *2 x 2 = 4 and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.* |  | |  |
| Year 4 - 6 | **Short division of 4-digit numbers by 1-digit numbers**  *Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.*  *They progress to calculations that require regrouping in the hundreds or tens columns.*  *Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.*  *The thought process of the traditional algorithm is as follows:*  *How many 4s in 8? 2*  *How many 4s in 5? 1 with 1 remaining so regroup.*  *How many 4s in 12? 3*  *How many 4s in 8? 2*  *Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.*  *How many 4s in 8000? 2000*  *How many 4s in 500?*  *100 with 1 remaining (illogical) The answer would be 125.*  *Area models are also useful representations, as seen with other strategies and exemplified for long division.* |  | | Begin with divisions that divide equally with no remainder.    Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. |
|  | |  |
| Year 5 and 6 | **Long division**  *Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.*  *If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.* |  | |  |

**2-digit** – a number with 2 digits like 23, 45, 12 or 60

**Glossary**

**3-digit** – a number with 3 digits like 123, 542, 903 or 561

**Addition facts** – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.

**Array** - An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

**Bridge to ten** – a strategy when using numberlines. Adding a number that takes you to the next ‘tens’ number.

**Bus Stop Method** - traditional method for division with a single digit divisor

**Concrete apparatus** – objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move. Dienes Base 10 (purple hundreds, tens and units blocks), Numicon, Cuisenaire rods are also referred to as concrete apparatus.

**Column chunking** – method of division involving taking chunks or groups of the divisor away from the larger number

**Decimal** **number** – a number with a decimal point

**Divisor** – the smaller number in a division calculation. The number in each group for chunking.

**Double** – multiply a number by 2

**Exchanging** – Moving a ‘ten’ or a ‘hundred’ from its column into the next column and splitting it up into ten ‘ones’ (or ‘units’) or ten ‘tens’ and putting it into a different column

**Expanded Multiplication** – a method for multiplication where each stage is written down and then added up at the end in a column

**Find the difference** – A method for subtraction involving counting up from the smaller to the larger number

**Grid method** – a method for multiplying two numbers together involving partitioning

**Half** - a number, shape or quantity divided into 2 equal parts

**Halve** – divide a number by 2

**Integer** - a number with no decimal point

**Inverse** – the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division

**Jigsaw numbers** – addition facts to 100, see number bonds to 100

**Long Multiplication** – column multiplication where only the significant figures are noted

**Number bonds to ten** – 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

**Number bonds to 100** – 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88

**Numberline** – a line either with numbers or without (a blank numberline). Children use this tool to help them count on for addition or subtraction and also in multiplication and division.

**Numberline Chunking** - method of division involving taking chunks or groups of the divisor away from the larger number

**Number sentence** – writing out a calculation with just the numbers in a line e.g. 2+4=6 or 35 ÷7 = 5 or 12 x 3 =36 or 32 – 5 = 27

**Partition** – split up a larger number into the hundreds, tens and units. e.g. 342 – 300 and 40 and 2

**Place Value** – knowing that in the number 342 – the ‘3’ means ‘3 hundreds’, the ‘4’ means ‘4 tens’ and the ‘2’ means ‘2’

**Quarter** - a number, shape or quantity divided into 4 equal parts

**Recombine** – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add the hundreds together, then add the tens to that total, and then add the units to that total

**Remainder** – a whole number left over after a division calculation

**Repeated addition** – repeatedly adding groups of the same size for multiplication

**Significant digit** – the digit in a number with the largest value. eg. in 34 – the most significant digit is the 3, as it has a value of ‘30’ and the ‘4’ only has a value of ‘4’

**Single digit** – a number with only one digit. These are always less than 10.

**Taking away** – a method for subtraction involving counting backwards from the larger to the smaller number

**Tens number** - a number in the ten times tables – 10,20,30,40, 50, etc.

**Unit** – another term for single digit numbers. The right hand column in column methods is the ‘units’ column. This should be replaced by ones